

## On the Numerical Evaluation of Two Infinite Products

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**Abstract.** A numerical evaluation of two infinite products of the type  $\prod_{n=0}^{\infty}(1 - aq^n)$ , which are important in some mathematical fields, is considered.

The numerical evaluation is based on a recursive formula of the type  $x_{n+1} = x_n f(y_n/x_n)$ ,  $y_{n+1} = x_{n+1} g(y_n/x_n)$ , and it is compared with a series expansion that was previously used for the computation.

Two tables of the infinite products are provided with twenty significant figures which check and extend existing data.

1. **Introduction.** The infinite product

$$(1.1) \quad P = \prod_{n=0}^{\infty} (1 - aq^n), \quad |a| < 1, \quad 0 \leq q < 1,$$

and its particular case, for  $a = q$ ,

$$(1.2) \quad Q = \prod_{n=1}^{\infty} (1 - q^n)$$

are very important in some mathematical fields (elliptic, hypergeometric, modular and partition functions); and therefore, it is very useful to have their numerical evaluation.

A table of the reciprocal of (1.1) has been built by L. Slater [1], [2] with seven significant figures for  $a = -0.90(0.05)0.95$  and  $q = 0.00(0.05)0.95$ . The table was constructed using the formula

$$(1.3) \quad P^{-1} = \sum_{n=0}^{\infty} \frac{a^n}{(q)_n},$$

where

$$(q)_n = (1 - q)(1 - q^2) \cdots (1 - q^n), \quad (q)_0 = 1.$$

L. Slater mentions some computing difficulties for  $0.89 < q < 1$  that can be avoided by using the logarithmic form:

$$\ln P = \ln \prod_{n=0}^{\infty} (1 - aq^n) = \sum_{n=0}^{\infty} \ln(1 - aq^n).$$

Except for the tables in [3] and a short table in [4], we know of no other tables.

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Received August 20, 1979.

1980 *Mathematics Subject Classification.* Primary 65D20.

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0025-5718/80/0000-0117/\$04.75

2. **The Algorithm.** L. Gatteschi has suggested [5] the iterative scheme

$$(2.1) \quad \begin{aligned} x_{n+1} &= x_n \frac{(q-1)x_n + (3-q)y_n}{2y_n}, \\ y_{n+1} &= x_{n+1} \frac{y_n}{qx_n + (1-q)y_n}, \end{aligned} \quad (n = 0, 1, 2, \dots).$$

If the initial values  $(x_0, y_0)$  satisfy the constraint

$$\frac{x_0}{y_0} = \frac{1 + 2a - q}{1 - q},$$

then the algorithm (2.1) has the limit function

$$(2.2) \quad X(x_0, y_0) = \lim_{n \rightarrow \infty} x_n = \lim_{n \rightarrow \infty} y_n = x_0 \cdot P$$

provided  $x_0 \neq 0$ , that is  $1 + 2a - q \neq 0$ . In the case  $x_0 = 0$ , that is,  $a = (q - 1)/2$ , the infinite product (1.1) can be replaced by

$$\prod_{n=0}^{\infty} (1 - a^*q^n) = \frac{P}{1 - a},$$

where  $a^* = aq$ . In this way we have

$$1 + 2a^* - q = (q - 1)^2 \neq 0,$$

and the given procedure applies.

A neat property of algorithm (2.1) is its bilateral convergence; that is, for appropriate initial values  $x_0$  and  $y_0$ ,  $\{x_n\}$  and  $\{y_n\}$  are upper and lower bounds, respectively, of the limit (2.2). Thus, the sequence  $\{t_n\}$ ,

$$t_n = \frac{1}{2}(x_n + y_n)$$

is an improved approximation over  $x_n$  and  $y_n$ .

3. **Related Infinite Products.** There are some cases that can be reduced to the form (1.1) and, therefore, computed with the algorithm (2.1). They are

$$(3.1) \quad \prod_{n=1}^{\infty} (1 + q^n),$$

$$(3.2) \quad \prod_{n=1}^{\infty} (1 - q^{2n}) = Q_0,$$

$$(3.3) \quad \prod_{n=1}^{\infty} (1 + q^{2n}) = Q_1,$$

$$(3.4) \quad \prod_{n=1}^{\infty} (1 + q^{2n-1}) = Q_2,$$

$$(3.5) \quad \prod_{n=1}^{\infty} (1 - q^{2n-1}) = Q_3.$$

The infinite product (3.1) is obtained from (1.1) by setting  $a = -q$ ; (3.2) and (3.3) can be reduced to (1.2) and (3.1), respectively, with the substitution  $q^2 = p$ ; the infinite products (3.4) and (3.5) can be reduced to (1.1) by putting  $q^2 = p$  and  $a = \mp q$ .

The infinite products (3.1) to (3.4) are related [7] since

$$Q_0 Q_3 = \prod_{n=1}^{\infty} (1 - q^n), \quad Q_1 Q_2 = \prod_{n=1}^{\infty} (1 + q^n), \quad Q_1 Q_2 Q_3 = 1.$$

A table of  $\ln Q_0$  has been published by F. W. Newman [3], [6]. This table has sixteen significant figures and has been computed for  $\rho = -\frac{1}{2} \ln q = 1.0(0.1)4.6$ .

**4. Numerical Evaluation.** The computation was done on the ELEA 6001 computer at the Institute of Numerical Analysis of the University of Turin. To avoid the loss of significant digits we used a floating-point arithmetic package with a twenty-three-digit mantissa.

The values given in Table 1 were found using both (1.3) and (2.1), and so may be considered exact to the number of places given.

Regarding the particular case (1.2), L. Slater published [1], [2] a table of numerical values of its reciprocal for  $q = 0.000(0.005)0.995$  with seven significant figures. We extended this table to twenty significant figures (unrounded) using (1.3) and (2.1) (Table 2).

As noted earlier, L. Slater reported difficulties in computing with (1.3) for  $0.89 < q < 1$  in view of slow convergence. For this range, we would suggest the algorithm [8]

$$(4.1) \quad \begin{aligned} z_{n+1} &= qz_n, & z_0 &= a, \\ w_{n+1} &= w_n(1 + z_n), & w_0 &= 1. \end{aligned}$$

Indeed we computed Table 1 in the above range using the latter scheme. Though convergence is slow, the algorithm is very simple and easy to do on a computer.

TABLE 1\*

$\prod_{n=0}^{\infty} (1 - aq^n)^{-1}$								
a -- 0.96		q	a -- 0.90					
0.51282	05128	20512	82051	0.00	0.52631	57894	73684	21052
0.48834	51138	97940	28694	0.05	0.50246	12990	45982	44736
0.46343	27507	17605	64411	0.10	0.47807	34415	23940	15592
0.43781	23931	19928	75637	0.15	0.45289	13391	11221	52303
0.41125	22409	55912	84825	0.20	0.42668	52772	05462	01320
0.38355	31987	57645	30821	0.25	0.39925	09577	21883	58089
0.35454	64766	04435	59754	0.30	0.37040	73268	71335	74446
0.32409	64642	45412	00188	0.35	0.33999	91418	31497	46554
0.29211	08044	44760	97276	0.40	0.30790	62342	95946	30345
0.25856	07580	42055	10322	0.45	0.27406	26656	61069	88889
0.22351	67597	49139	78698	0.50	0.23849	09560	58490	70791
0.18720	67014	44888	03389	0.55	0.20135	96423	32958	95391
0.15010	77302	28175	08509	0.60	0.16307	66869	84928	32817
0.11308	43065	22753	75866	0.65	0.12443	55104	31227	86327
(-1) 0.77578	06152	42506	58405	0.70	(-1) 0.86828	08307	09860	15037
0.45811	60117	88387	19435	0.75	0.52505	02808	09887	26521
0.20808	67988	24542	67178	0.80	0.24711	31010	04001	32184
(-2) 0.55915	19591	45526	17708	0.85	(-2) 0.70445	61046	11047	15087
(-3) 0.40438	24523	96932	29328	0.90	(-3) 0.57333	90599	86423	00921
(-6) 0.15340	94495	38833	28525	0.95	(-6) 0.30993	01974	43531	01187

\*The notation  $(k)0.n$  means  $0.n \cdot 10^k$ . The number  $(k)$  is omitted after its first appearance.

TABLE I (continued)

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$s - - 0,85$		$q$	$s - - 0,80$
0.54054	05405 40540 54054	0.00	0.55555 55555 55555 55555
0.51734	67693 26007 18057	0.05	0.53306 56781 48909 17967
0.49352	89376 47059 25499	0.10	0.50986 74742 13704 01665
0.46883	53464 76974 41457	0.15	0.48571 75404 17841 20160
0.44303	77997 72802 33317	0.20	0.46038 87252 73508 45236
0.41592	63985 82241 83896	0.25	0.43366 52419 13648 63963
0.38730	73793 90848 47029	0.30	0.40534 03084 65613 47139
0.35700	51427 48221 06846	0.35	0.37521 74089 70190 73088
0.32487	04168 03125 86063	0.40	0.34311 70589 23366 77960
0.29079	77891 09402 50245	0.45	0.30889 22772 51210 83999
0.25475	79805 16655 49734	0.50	0.27245 82232 47789 63740
0.21685	37068 31511 03491	0.55	0.23384 52741 75779 73332
0.17741	32315 31855 94000	0.60	0.19329 10067 44624 74157
0.13714	23283 82769 96152	0.65	0.15139 55319 72976 14432
(-1) 0.97358	41024 26165 00050	0.70	0.10937 34362 85708 03711
0.60307	04662 89185 41379	0.75	(-1) 0.69424 97836 62197 41081
0.29424	81330 59050 41292	0.80	0.35135 46121 64644 57410
(-2) 0.89067	65702 62081 70349	0.85	0.11302 89435 09483 65076
(-3) 0.81719	15347 76530 17899	0.90	(-2) 0.11711 68622 48648 49696
(-6) 0.63273	66049 97645 13253	0.95	(-5) 0.13058 86440 92435 14373
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$s - - 0,75$		$q$	$s - - 0,70$
0.57142	85714 28571 42857	0.00	0.58823 52941 17647 05882
0.54968	95112 03154 16603	0.05	0.56729 81647 36601 07564
0.52716	50910 56251 22451	0.10	0.54550 67806 17605 84953
0.50361	94320 78127 19421	0.15	0.52263 21527 78269 92442
0.47882	60716 03702 37418	0.20	0.49844 83000 78858 60653
0.45256	31900 94962 71605	0.25	0.47272 74053 79523 35616
0.42461	08555 80299 71591	0.30	0.44523 64801 11645 33728
0.39475	12917 79423 51823	0.35	0.41573 64238 04953 76122
0.36277	39444 35013 48943	0.40	0.38398 50929 36475 42700
0.32848	84393 82975 48635	0.45	0.34974 72812 76204 13911
0.29175	08526 33127 87607	0.50	0.31281 68543 90161 50140
0.25251	28017 52614 52165	0.55	0.27306 05465 25786 26108
0.21090	98829 34511 67071	0.60	0.23050 06200 98882 48466
0.16741	72547 46988 52910	0.65	0.18546 69682 19968 66133
0.12311	49701 16369 38978	0.70	0.13887 00482 21700 81680
(-1) 0.80109	76637 49133 92186	0.75	(-1) 0.92666 26746 51219 23342
0.42076	74583 13568 15480	0.80	0.50542 56686 02419 46118
0.14398	87528 52298 61285	0.85	0.18416 49689 47014 08797
(-2) 0.16880	85320 22196 33333	0.90	(-2) 0.24476 62341 88327 11677
(-5) 0.27258	31037 69899 42469	0.95	(-5) 0.57571 01472 22318 45462

TABLE 1 (continued)

<hr/>								
$a = -0.65$		$q$	$a = -0.60$					
<hr/>								
0.60606	06060	06060	06060	0.00	0.62500	00000	00000	00000
0.58598	12198	89650	09276	0.05	0.60583	94559	65945	55718
0.56498	78445	70545	31909	0.10	0.58571	55048	07954	15701
0.54285	79126	95702	24824	0.15	0.56441	17330	94291	36563
0.51936	59018	01117	47189	0.20	0.54170	32816	93194	55652
0.49427	82573	81389	57009	0.25	0.51735	13989	64967	74929
0.46734	93119	69567	37327	0.30	0.49109	84723	82213	55801
0.43831	89356	06211	92988	0.35	0.46266	40926	05556	64070
0.40691	33201	97464	74327	0.40	0.43174	32886	88594	91699
0.37285	15198	24459	38151	0.45	0.39800	91832	50937	72781
0.33586	26567	68262	72155	0.50	0.36112	44748	27880	96287
0.29572	30262	58684	41017	0.55	0.32077	03921	56465	87681
0.25233	04963	70097	57817	0.60	0.27671	01341	62546	86471
0.20584	87686	00389	61789	0.65	0.22892	03030	46508	12154
0.15698	05978	69762	82494	0.70	0.17785	54474	56645	90907
0.10746	62593	12035	51286	0.75	0.12496	49610	59195	53394
(-1) 0.60904	47029	60254	63782	0.80	(-1) 0.73634	30520	48744	27090
0.23653	76948	79666	64638	0.85	0.30513	32058	63454	30601
(-2) 0.35710	89812	28226	61225	0.90	(-2) 0.52439	69952	91619	07364
(-4) 0.12309	33409	24484	27871	0.95	(-4) 0.26657	57123	36612	48540
<hr/>								
$a = -0.55$		$q$	$a = -0.50$					
<hr/>								
0.64516	12903	22580	64516	0.00	0.66666	66666	66666	66666
0.62698	66565	94276	74371	0.05	0.64955	17771	22957	57727
0.60781	08262	42700	26562	0.10	0.63141	10241	72910	88067
0.58742	35138	73695	64935	0.15	0.61204	05146	82388	91606
0.56560	10074	28423	33072	0.20	0.59121	85084	15434	15641
0.54210	02447	53171	15679	0.25	0.56869	89462	65428	50595
0.51665	28461	42616	48492	0.30	0.54420	44108	72969	52347
0.48895	94477	90748	43304	0.35	0.51741	86311	17375	03073
0.45868	51601	77794	35748	0.40	0.48797	89922	84992	37138
0.42545	79317	73019	35815	0.45	0.45547	02700	00444	77035
0.38387	35418	63262	51976	0.50	0.41942	24417	95107	59770
0.34851	49305	86664	89643	0.55	0.37931	89717	40875	89343
0.30400	18191	78186	42074	0.60	0.33462	96910	55539	27819
0.25510	37581	31102	49157	0.65	0.28489	94573	51548	65961
0.20198	53293	13560	43658	0.70	0.22996	17203	56730	56915
0.14572	21271	39039	22806	0.75	0.17042	91301	46612	27893
(-1) 0.89334	17196	52021	57765	0.80	0.10877	62467	57241	51070
0.39542	25329	50003	64343	0.85	(-1) 0.51488	48142	43322	97069
(-2) 0.77527	64884	31176	99305	0.90	0.11543	21208	94741	75815
(-4) 0.58507	08846	02925	52763	0.95	(-3) 0.13021	57649	70707	72174

TABLE 1 (continued)

$a = -0.45$				$q$	$a = -0.40$			
0.68965	51724	13793	10344	0.00	0.71428	57142	85714	28571
0.67368	15604	40734	39800	0.05	0.69954	37133	30853	38771
0.65667	22490	38967	09849	0.10	0.68377	29679	20348	76143
0.63843	03474	23029	55562	0.15	0.66678	46225	00871	83581
0.61873	73334	89194	07573	0.20	0.64836	51070	04850	79486
0.59734	59836	37245	10800	0.25	0.62826	85281	52047	47142
0.57397	22580	04962	60436	0.30	0.60620	74786	87176	75931
0.54828	59299	68149	96077	0.35	0.58184	18592	26316	45131
0.51990	00602	27015	23693	0.40	0.55476	53370	02145	73373
0.48835	98700	85656	31096	0.45	0.52448	92878	20183	25988
0.45313	28165	11132	34503	0.50	0.49042	48018	46021	34733
0.41360	45624	37234	47983	0.55	0.45186	53656	73286	27052
0.36909	22951	35395	42375	0.60	0.40797	81325	17559	81762
0.31890	27800	40769	68946	0.65	0.35782	53364	62256	49165
0.26250	06357	17292	94557	0.70	0.30047	28571	38331	53730
0.19994	39376	94712	85431	0.75	0.23533	65654	07558	23420
0.13295	60753	78410	95545	0.80	0.16316	43701	35862	26606
(-1) 0.67380	93063	18963	02605	0.85	(-1) 0.88644	58991	48017	04342
0.17314	81813	42112	63723	0.90	0.26175	20698	25563	30007
(-3) 0.29408	43514	86895	73128	0.95	(-3) 0.67443	92378	88779	68069

  

$a = -0.35$				$q$	$a = -0.30$			
0.74074	07407	40740	74074	0.00	0.76923	07692	30769	23076
0.72733	07886	81705	26539	0.05	0.75726	49635	92354	54837
0.71291	80974	82879	00706	0.10	0.74434	40906	90172	50808
0.69732	33731	96733	91859	0.15	0.73030	05457	39877	57860
0.68034	03644	23902	40384	0.20	0.71493	85042	03801	42173
0.66172	77755	22175	10040	0.25	0.69802	55113	84254	25923
0.64119	91391	84075	41661	0.30	0.67928	16394	64979	92916
0.61840	99784	37708	58683	0.35	0.65836	52928	92889	57506
0.59294	14346	59294	14638	0.40	0.63485	44027	61273	60650
0.56427	94382	13574	96115	0.45	0.60822	13107	64990	28815
0.53178	86524	63274	55238	0.50	0.57779	91597	24609	54060
0.49468	13908	15771	24682	0.55	0.54273	73494	33252	85152
0.45198	50443	80631	64424	0.60	0.50194	45365	83308	91728
0.40252	16353	40798	93396	0.65	0.45402	28924	20339	37858
0.34494	25805	35095	85373	0.70	0.39721	71623	75969	97124
0.27794	85340	20523	68517	0.75	0.32947	11915	70581	52788
0.20108	44203	79022	68151	0.80	0.24892	52493	24757	96187
0.11726	68905	93982	92726	0.85	0.15604	03677	13823	62180
(-1) 0.39894	83101	17398	11403	0.90	(-1) 0.61331	86800	49377	90243
(-2) 0.15718	51624	41564	37190	0.95	(-2) 0.37259	96584	91971	75521

TABLE 1 (continued)

$a = -0.25$				$q$	$a = -0.20$			
0.80000	00000	00000	00000	0.00	0.83333	33333	33333	33333
0.78960	39642	23525	77268	0.05	0.82464	84718	61473	07648
0.77832	52443	33518	25074	0.10	0.81518	15834	52515	01778
0.76601	07722	80832	49421	0.15	0.80479	78516	97679	88418
0.75247	91685	83614	74093	0.20	0.79333	54700	58117	92158
0.73751	22541	53801	12550	0.25	0.78059	74354	44567	55133
0.72084	38427	08293	64427	0.30	0.76634	04927	38951	42623
0.70214	46833	83459	82953	0.35	0.75025	99578	11618	97207
0.68100	19073	21864	83720	0.40	0.73196	84884	27488	55707
0.65689	05594	03043	41044	0.45	0.71096	58150	84960	88934
0.62913	36626	92661	39656	0.50	0.68659	47225	84429	88626
0.59684	56929	14045	82336	0.55	0.65797	57486	54664	33143
0.55885	25210	31233	45843	0.60	0.62390	84639	25533	06712
0.51358	01981	55609	94905	0.65	0.58272	00575	20497	59975
0.45891	16156	67810	09073	0.70	0.53203	28769	34582	30019
0.39204	96587	79577	26550	0.75	0.46842	18839	09874	95396
0.30960	50916	23115	34852	0.80	0.38700	63645	28894	18565
0.20891	97850	23059	37556	0.85	0.28155	27761	52169	06964
(-1) 0.95149	14941	99489	67693	0.90	0.14903	85392	50572	86043
(-2) 0.89915	12941	59165	71018	0.95	(-1) 0.22111	43626	25564	32416

  

$a = -0.15$				$q$	$a = -0.10$			
0.86956	52173	91304	34782	0.00	0.90909	09090	90909	09090
0.86275	14609	98656	78227	0.05	0.90433	00841	63850	03301
0.85528	88025	70030	14144	0.10	0.89909	09181	82726	45446
0.84706	54403	63857	76920	0.15	0.89329	06586	88780	71214
0.83794	56253	06479	15481	0.20	0.88682	77626	23151	23462
0.82776	22383	43948	39887	0.25	0.87957	59394	12866	45999
0.81630	66163	09454	28521	0.30	0.87137	58606	27797	62083
0.80331	43243	60769	15525	0.35	0.86202	33769	85695	12190
0.78844	48382	26206	71701	0.40	0.85125	23841	52331	04651
0.77125	18656	84458	66041	0.45	0.83870	92661	30010	47325
0.75113	89076	41992	40278	0.50	0.82391	36671	01315	86352
0.72729	08754	37917	55544	0.55	0.80619	62712	98887	56967
0.69856	56512	89041	82304	0.60	0.78459	53047	20531	19851
0.66331	64368	84673	18503	0.65	0.75768	50559	08643	69290
0.61909	08216	88361	11764	0.70	0.72324	58313	78623	05316
0.56210	62606	91849	94475	0.75	0.67765	71018	53959	00416
0.48633	32200	81862	84878	0.80	0.61461	78461	85306	09883
0.38207	63939	81564	92875	0.85	0.52232	59130	74359	35772
0.23583	89323	20549	08249	0.90	0.37725	04151	70770	49557
(-1) 0.55471	73896	36748	02345	0.95	0.14214	20228	60639	77132

TABLE 1 (continued)

$a = 0.05$	$q$	$a = 0.00$
0.95238 09523 80952 38095	0.00	1.00000 00000 00000 00000
0.94988 09524 18154 99439	0.05	1.00000 00000 00000 00000
0.94711 65362 59366 32100	0.10	1.00000 00000 00000 00000
0.94404 15853 61488 68635	0.15	1.00000 00000 00000 00000
0.94059 89607 29518 42616	0.20	1.00000 00000 00000 00000
0.93671 69225 33481 06160	0.25	1.00000 00000 00000 00000
0.93230 40786 78279 01396	0.30	1.00000 00000 00000 00000
0.92724 20998 07847 48805	0.35	1.00000 00000 00000 00000
0.92137 49526 59732 81488	0.40	1.00000 00000 00000 00000
0.91449 25398 09694 27319	0.45	1.00000 00000 00000 00000
0.90630 50338 11447 44987	0.50	1.00000 00000 00000 00000
0.89640 10889 21096 97231	0.55	1.00000 00000 00000 00000
0.88417 67514 46978 09332	0.60	1.00000 00000 00000 00000
0.86870 79829 49920 57123	0.65	1.00000 00000 00000 00000
0.84850 68494 15947 41408	0.70	1.00000 00000 00000 00000
0.82101 59662 46718 23206	0.75	1.00000 00000 00000 00000
0.78144 49263 31551 98890	0.80	1.00000 00000 00000 00000
0.71969 07680 95673 74200	0.85	1.00000 00000 00000 00000
0.61044 29682 06404 01357	0.90	1.00000 00000 00000 00000
0.37252 04021 02294 63534	0.95	1.00000 00000 00000 00000

$a = 0.05$	$q$	$a = 0.10$
1.0526 31578 94736 84210	0.00	1.1111 11111 11111 11111
1.0554 08622 01611 38698	0.05	1.1169 88524 69636 09159
1.0585 09215 84731 23194	0.10	1.1235 82754 84865 25111
1.0619 91307 83690 62092	0.15	1.1310 24439 46357 19081
1.0659 27930 73374 00789	0.20	1.1394 79498 49957 11546
1.0704 12364 86185 45509	0.25	1.1491 61531 45269 42985
1.0755 65598 46169 24377	0.30	1.1603 49929 39988 45512
1.0815 47367 43579 93809	0.35	1.1734 16992 49331 20598
1.0885 72944 98188 48916	0.40	1.1888 69711 53357 95570
1.0969 39499 58050 65320	0.45	1.2074 16337 50273 19105
1.1070 69041 24149 40585	0.50	1.2300 76712 49054 89539
1.1195 81547 06404 81151	0.55	1.2583 73925 79307 23042
1.1354 26256 74318 91581	0.60	1.2946 86721 57303 60095
1.1561 33285 22432 13422	0.65	1.3429 44670 67102 26121
1.1843 37090 83643 09387	0.70	1.4101 27492 61740 60080
1.2249 88658 87449 55968	0.75	1.5099 01719 78308 87032
1.2886 05141 73394 74572	0.80	1.6729 98480 14678 68878
1.4020 69280 83973 59149	0.85	1.9849 71804 84259 33002
1.6598 66870 70182 10585	0.90	2.7944 17750 57776 93310
2.7541 84424 67640 37144	0.95	7.7971 90522 73662 93982



TABLE 1 (continued)

$a = 0.15$				$q$	$a = 0.20$			
1.1764	70588	23529	41176	0.00	1.2500	00000	00000	00000
1.1858	28875	73873	59930	0.05	1.2632	91136	03396	87155
1.1963	80048	63130	36692	0.10	1.2783	50408	88484	86645
1.2083	45833	01930	45732	0.15	1.2955	13659	90565	29558
1.2220	09897	18892	44181	0.20	1.3152	13555	73534	52193
1.2377	40200	80893	78394	0.25	1.3380	15456	07731	81886
1.2560	21983	89364	57911	0.30	1.3646	70882	04803	14710
1.2775	07712	37750	29580	0.35	1.3961	99342	34780	08807
1.3030	94994	84385	30273	0.40	1.4340	17524	63479	87065
1.3340	52488	51591	06883	0.45	1.4801	50914	98552	80809
1.3722	31989	26206	89815	0.50	1.5375	95890	61318	61924
1.4204	43907	24467	57039	0.55	1.6109	72542	79888	10311
1.4831	63385	46534	45152	0.60	1.7077	79145	45403	18288
1.5679	61648	77030	99592	0.65	1.8410	02103	46199	03624
1.6886	99010	58094	87985	0.70	2.0351	28606	56269	72797
1.8736	11626	98368	75530	0.75	2.3420	14533	72960	94413
2.1897	67143	21739	69131	0.80	2.8915	99965	62774	63487
2.8398	50840	48261	38037	0.85	4.1094	46428	25274	57262
4.7768	03074	17835	60661	0.90	8.3016	11171	67174	21923
(1) 2.2737	82765	02993	41185	0.95	(1) 6.8464	27220	23748	99802

  

$a = 0.25$				$q$	$a = 0.30$			
1.3333	33333	33333	33333	0.00	1.4285	71428	57142	85714
1.3510	99824	08022	55780	0.05	1.4514	72179	43296	38091
1.3713	29654	65457	10285	0.10	1.4776	78151	35210	75904
1.3945	01741	01610	74995	0.15	1.5078	47998	75243	22461
1.4212	37240	97832	01052	0.20	1.5428	41634	85978	31048
1.4523	53642	44959	70158	0.25	1.5837	98774	53308	24334
1.4889	46165	74960	37527	0.30	1.6322	58243	36999	51710
1.5325	13646	02529	07457	0.35	1.6903	44374	08995	53440
1.5851	59615	37810	60760	0.40	1.7610	68177	93168	46417
1.6499	26069	58541	13295	0.45	1.8488	33775	54051	89044
1.7313	73309	72753	18057	0.50	1.9603	31413	23152	71165
1.8366	44012	27649	32647	0.55	2.1062	04903	45919	73240
1.9775	51180	62045	93537	0.60	2.3043	91252	08320	12406
2.1750	33098	04056	14892	0.65	2.5874	24334	63460	32902
2.4697	41257	53753	12709	0.70	3.0203	31203	28047	24107
2.9511	19248	01226	17016	0.75	3.7517	42531	86055	50132
3.8554	66620	83699	51316	0.80	5.1956	03851	21939	96382
6.0209	94045	44981	11329	0.85	8.9428	29572	86479	84739
(1) 1.4689	26595	95670	77246	0.90	(1) 2.6508	51358	61582	81947
(2) 2.1343	83768	49676	95541	0.95	(2) 6.9111	11388	37509	39355

TABLE 1 (continued)

$a = 0.35$				$q$	$a = 0.40$			
1.5384	61538	46153	84615	0.00	1.6666	66666	66666	66666
1.5673	07670	82425	47900	0.05	1.7024	72258	34210	68748
1.6004	82560	10111	88945	0.10	1.7438	58461	78095	61730
1.6388	71425	65130	61006	0.15	1.7919	97537	66180	78014
1.6836	37487	65296	86013	0.20	1.8484	39850	06919	34943
1.7363	33294	39318	17340	0.25	1.9152	69219	08782	38309
1.7990	71379	27433	58325	0.30	1.9953	43216	72361	60700
1.8747	93574	24079	15998	0.35	2.0926	76179	35897	86649
1.9677	11096	04388	33167	0.40	2.2130	72148	31393	09394
2.0840	54114	69020	74294	0.45	2.3652	16735	55613	92793
2.2334	13106	16052	06632	0.50	2.5626	59817	68864	36540
2.4312	86677	30319	98159	0.55	2.8276	46211	80808	97415
2.7042	88749	72506	17621	0.60	3.1991	03849	79422	83780
3.1018	17626	79326	83332	0.65	3.7509	84737	84618	78869
3.7255	24819	53617	60116	0.70	4.6399	97261	65819	76574
4.8167	99926	67822	75547	0.75	6.2529	04219	76759	16887
7.0845	88098	79891	18002	0.80	9.7883	81090	97072	59030
(1) 1.3484	08581	07929	31895	0.85	(1) 2.0674	94521	97117	98728
4.8885	41248	24194	29567	0.90	9.2342	66168	24035	13228
(3) 2.3328	73757	01883	27550	0.95	(3) 8.2449	78328	78206	53614
$a = 0.45$				$q$	$a = 0.50$			
1.8181	81818	18181	81818	0.00	2.0000	00000	00000	00000
1.8622	37707	70797	56442	0.05	2.0539	84493	12120	02018
1.9134	18048	75896	50222	0.10	2.1170	18131	69462	46389
1.9732	61347	78137	53184	0.15	2.1911	12124	32545	41035
2.0438	15525	53665	16186	0.20	2.2789	58996	99914	23093
2.12 8	52653	95114	99271	0.25	2.3842	31029	03137	17241
2.2292	04087	81195	61129	0.30	2.5120	43967	78729	15822
2.3532	99979	55747	91393	0.35	2.6697	13771	99777	98296
2.5080	70889	32174	11548	0.40	2.8680	35049	26959	74131
2.7055	23059	49633	98626	0.45	3.1235	43499	40440	09568
2.9646	41089	13531	07751	0.50	3.4627	46619	45506	36115
3.3170	92405	78051	31925	0.55	3.9305	81856	72587	09667
3.8193	64989	98207	81260	0.60	4.6087	82504	16640	24812
4.5813	22084	34451	65769	0.65	5.6601	58066	40996	11668
5.8427	94541	07831	03240	0.70	7.4510	49639	07235	20233
8.2193	35835	68342	66003	0.75	(1) 1.0960	08612	61523	57932
(1) 1.3726	27076	71977	96100	0.80	1.9576	76218	19414	51806
3.2302	34271	82002	86106	0.85	5.1556	73718	82367	39143
(2) 1.7917	63684	87150	02940	0.90	(2) 3.5835	27369	74300	05881
(4) 3.0668	66268	83405	82501	0.95	(5) 1.2081	89630	34500	55827

TABLE 1 (continued)

$a = 0.55$				$q$	$a = 0.60$			
2.2222	22222	22222	22222	0.00	2.5000	00000	00000	00000
2.2883	73302	00020	52725	0.05	2.5813	95168	28014	88214
2.3660	08817	58384	13597	0.10	2.6774	13071	38009	79240
2.4577	54655	24133	69334	0.15	2.7914	96461	28933	29955
2.5671	52308	69241	96440	0.20	2.9283	19863	84171	31986
2.6990	65178	40321	72295	0.25	3.0943	50502	02234	45985
2.8603	24138	66183	09836	0.30	3.2987	47693	26906	81863
3.0607	87859	54031	89266	0.35	3.5548	55246	90232	06685
3.3151	55306	49618	57078	0.40	3.8827	82169	47867	48129
3.6462	15094	29418	53347	0.45	4.3140	90475	71808	62467
4.0910	12835	86667	41030	0.50	4.9008	28533	07881	77912
4.7133	96949	21371	18651	0.55	5.7342	34531	09803	31897
5.6318	26552	57273	02797	0.60	6.9870	71297	85669	30664
7.0881	07646	87932	21200	0.65	9.0209	66372	31586	86709
9.6434	75962	19721	80270	0.70	(1) 1.2702	51046	17801	22559
(1) 1.4860	07293	62960	80096	0.75	2.0548	33958	92085	66500
2.8471	85491	68640	26066	0.80	4.2369	88779	91418	62092
8.4328	12665	75086	90042	0.85	(2) 1.4192	60156	66665	06228
(2) 7.4191	43124	48951	18123	0.90	(3) 1.5987	44986	67653	85855
(5) 5.0801	10448	77838	55286	0.95	(6) 2.3022	03662	21114	13533

  

$a = 0.65$				$q$	$a = 0.70$			
2.8571	42857	14285	71428	0.00	3.3333	33333	33333	33333
2.9581	78862	82347	19552	0.05	3.4606	05674	24175	04976
3.0779	83044	77118	09455	0.10	3.6123	05233	79526	60651
3.2211	06907	00060	87578	0.15	3.7945	35423	53678	58490
3.3937	73054	62356	37721	0.20	4.0156	99529	23843	76199
3.6046	56295	41693	48454	0.25	4.2876	04446	62797	23711
3.8661	47734	03985	72645	0.30	4.6272	63928	91697	30772
4.1964	74290	48009	56981	0.35	5.0599	54578	89144	92103
4.6234	07300	70587	61516	0.40	5.6246	28115	78518	83489
5.1910	90289	39476	00618	0.45	6.3840	14269	40646	53016
5.9734	01367	72236	60017	0.50	7.4447	10353	86840	22108
7.1021	25083	88881	39371	0.55	9.0003	32736	69455	84512
8.8321	04596	60979	60300	0.60	(1) 1.1433	55088	42893	47054
(1) 1.1710	80960	45464	16345	0.65	1.5588	92169	84018	72206
1.7094	31065	40022	88560	0.70	2.3631	78053	18030	01521
2.9098	30011	05796	63179	0.75	4.2444	24159	60778	30147
6.4811	26704	92203	45707	0.80	(2) 1.0255	82448	20100	54808
(2) 2.4711	41232	77750	40788	0.85	4.4844	84773	60291	40549
(3) 3.6115	28274	57233	49745	0.90	(3) 8.6359	01238	45181	36648
(7) 1.1387	32763	39638	29997	0.95	(7) 6.2521	06721	10298	85754

TABLE I (continued)

$a = 0.75$				$q$	$a = 0.80$			
4.0000	00000	00000	00000	0.00	5.0000	00000	00000	00000
4.1640	61926	70468	69780	0.05	5.2193	20274	46375	96932
4.3606	35425	67430	52907	0.10	5.4834	85315	99870	45871
4.5980	91600	18076	05836	0.15	5.8043	95414	65284	96740
4.8880	39588	75569	76724	0.20	6.1986	77191	79977	30564
5.2469	22619	40004	91256	0.25	6.6900	77280	38659	09434
5.6986	51150	15112	86009	0.30	7.3134	51378	28953	95637
6.2791	14683	06183	04322	0.35	8.1216	77720	47679	01534
7.0442	72711	72673	85669	0.40	9.1982	16424	09058	41729
8.0854	74858	56563	77912	0.45	(1) 1.0681	24568	18346	27078
9.5604	31333	82782	36276	0.50	1.2813	29908	84432	18270
(1) 1.1761	03368	89302	39072	0.55	1.6052	17774	01093	69541
1.5277	45995	99283	12504	0.60	2.1345	87442	47659	29175
2.1449	51413	07347	55048	0.65	3.0911	38117	03870	26315
3.3838	81356	59264	94827	0.70	5.0871	89870	14448	31539
6.4329	20560	32770	26281	0.75	(2) 1.0274	16979	46042	83250
(2) 1.6947	95511	96567	44837	0.80	2.9691	15980	84233	49162
8.5744	98327	74587	87626	0.85	(3) 1.7568	74208	44566	37885
(4) 2.2164	01833	67324	02847	0.90	(4) 6.2359	83165	99350	63648
(8) 3.9010	47438	70452	67581	0.95	(9) 2.8645	70401	11944	57627

  

$a = 0.85$				$q$	$a = 0.90$			
6.6666	66666	66666	66666	0.00	(1) 1.0000	00000	00000	00000
6.9781	83585	42475	71635	0.05	1.0496	06047	54403	40496
7.3553	82360	42146	41595	0.10	1.1099	90918	16529	83981
7.8162	27926	30551	53708	0.15	1.1841	93405	24762	01307
8.3860	12986	78757	94772	0.20	1.2765	25647	04743	69066
9.1011	72883	32611	11005	0.25	1.3932	54877	69576	14366
(1) 1.0015	70988	11685	20877	0.30	1.5437	64073	11941	07549
1.1212	45990	98346	64405	0.35	1.7426	12808	20442	44850
1.2823	83834	57850	95571	0.40	2.0133	85992	10657	72735
1.5072	33110	66195	93329	0.45	2.3963	29028	79288	60602
1.8355	27794	21535	85711	0.50	2.9646	41089	13531	07751
2.3438	37360	31530	65708	0.55	3.8625	05380	99203	05620
3.1947	66894	92092	47631	0.60	5.4044	39659	02846	01368
4.7807	63786	09443	89459	0.65	8.3755	80089	58247	06148
8.2308	72381	22807	28979	0.70	(2) 1.5136	24498	69329	01630
(2) 1.7738	33532	87253	71733	0.75	3.4997	54096	80566	75426
5.6644	83827	15814	46269	0.80	(3) 1.2464	54333	21566	73656
(3) 3.9730	98640	63737	13114	0.85	(4) 1.0545	71128	21699	83252
(5) 1.9934	36457	96594	51953	0.90	(5) 7.7756	42033	59582	18102
(10) 2.6198	88339	10158	30132	0.95	(11) 3.3198	06489	42661	37463

TABLE 1 (continued)

$a = 0.95$					$q$
(1)	2.0000	00000	00000	00000	0.00
	2.1049	99404	83629	18095	0.05
	2.2334	97941	88780	72063	0.10
	2.3923	24201	05426	79082	0.15
	2.5912	47116	20824	30006	0.20
	2.8445	95794	21442	24281	0.25
	3.1740	48726	42920	70540	0.30
	3.6136	51005	06275	09066	0.35
	4.2193	24579	69701	21867	0.40
	5.0880	62231	50337	20179	0.45
	6.3997	20823	57314	08050	0.50
	8.5169	38072	40892	51853	0.55
(2)	1.2253	86366	04418	48702	0.60
	1.9718	39887	53334	36821	0.65
	3.7559	06016	64008	53193	0.70
	9.3810	09488	56309	79815	0.75
(3)	3.7717	86542	47821	52420	0.80
(4)	3.9424	77432	01172	37355	0.85
(6)	4.5197	01741	04081	45196	0.90
(12)	7.6292	45417	67438	93757	0.95

TABLE 2

$$\prod_{n=1}^{\infty} (1 - q^n)^{-1}$$

$q$					$q$
0.000	1.0000	00000	00000	00000	0.200
0.005	1.0050	50378	14704	80555	0.205
0.010	1.0102	03050	71115	22304	0.210
0.015	1.0154	60383	56854	23272	0.215
0.020	1.0208	24823	12377	90018	0.220
0.025	1.0262	98899	26495	01275	0.225
0.030	1.0318	85228	46210	05252	0.230
0.035	1.0375	86517	01616	35943	0.235
0.040	1.0434	05564	46610	87734	0.240
0.045	1.0493	45267	16249	34265	0.245
0.050	1.0554	08622	01611	38698	0.250
0.055	1.0615	98730	43099	01598	0.255
0.060	1.0679	18802	43149	54887	0.260
0.065	1.0743	72160	99405	63270	0.265
0.070	1.0809	62246	59450	67573	0.270
0.075	1.0876	92621	98288	29779	0.275
0.080	1.0945	66977	19819	51837	0.280
0.085	1.1015	89134	83651	77086	0.285
0.090	1.1087	63055	58659	85256	0.290
0.095	1.1160	92844	04811	03524	0.295
					1.3152
					1.3274
					1.3398
					1.3527
					1.3658
					1.3793
					1.3931
					1.4073
					1.4219
					1.4369
					1.4523
					1.4681
					1.4844
					1.5011
					1.5182
					1.5359
					1.5541
					1.5728
					1.5920
					1.6118
					1.3152
					1.3274
					1.3398
					1.3527
					1.3658
					1.3793
					1.3931
					1.4073
					1.4219
					1.4369
					1.4523
					1.4681
					1.4844
					1.5011
					1.5182
					1.5359
					1.5541
					1.5728
					1.5920
					1.6118
					1.3152
					1.3274
					1.3398
					1.3527
					1.3658
					1.3793
					1.3931
					1.4073
					1.4219
					1.4369
					1.4523
					1.4681
					1.4844
					1.5011
					1.5182
					1.5359
					1.5541
					1.5728
					1.5920
					1.6118

TABLE 2 (continued)

q		q	
0.100	1.1235 82754 84865 25111	0.300	1.6322 58243 36999 51710
0.105	1.1312 37199 07667 05390	0.305	1.6532 77259 63023 18947
0.110	1.1390 60751 04859 50054	0.310	1.6749 37423 58363 71664
0.115	1.1470 58155 42971 82087	0.315	1.6972 65210 41760 28649
0.120	1.1552 34334 72963 40910	0.320	1.7202 88489 24572 84066
0.125	1.1635 94397 19447 01027	0.325	1.7440 36611 68255 89379
0.130	1.1721 43645 11964 77978	0.330	1.7685 40507 01047 13488
0.135	1.1808 87583 60853 02820	0.335	1.7938 32784 50048 81567
0.140	1.1898 31929 80405 86813	0.340	1.8199 47843 50275 91232
0.145	1.1989 82622 62235 68280	0.345	1.8469 21991 98217 94607
0.150	1.2083 45833 01930 45732	0.350	1.8747 93574 24079 15998
0.155	1.2179 27974 82325 77840	0.355	1.9036 03108 64201 23094
0.160	1.2277 35716 16944 05429	0.360	1.9333 93436 23320 59842
0.165	1.2377 75991 57406 69703	0.365	1.9642 09881 25366 96704
0.170	1.2480 56014 68898 24493	0.370	1.9961 00424 61581 77531
0.175	1.2585 83291 78056 43723	0.375	2.0291 15891 55951 51566
0.180	1.2693 65635 97981 00572	0.380	2.0633 10154 80453 92641
0.185	1.2804 11182 35398 52532	0.385	2.0987 40354 66568 03876
0.190	1.2917 28403 85393 37728	0.390	2.1354 67137 75087 15389
0.195	1.3033 26128 19518 26019	0.395	2.1735 54916 03708 24813
0.400	2.2130 72148 31393 09394	0.600	6.9870 71297 85669 30664
0.405	2.2540 91646 20380 98901	0.605	7.3111 13832 45909 25328
0.410	2.2966 90907 21295 60596	0.610	7.6597 97993 44784 35519
0.415	2.3409 52477 54390 29357	0.615	8.0355 97396 86579 37628
0.420	2.3869 64347 71031 63732	0.620	8.4412 92336 03937 95667
0.425	2.4348 20384 34506 49372	0.625	8.8800 14968 82778 02379
0.430	2.4846 20801 98701 15185	0.630	9.3553 02222 51977 83580
0.435	2.5364 72679 07774 45400	0.635	9.8711 57917 52719 37583
0.440	2.5904 90522 90358 79303	0.640	(1) 1.0432 12593 92070 68538
0.445	2.6467 96888 78916 86687	0.645	1.1043 37669 51521 17213
0.450	2.7055 23059 49633 98626	0.650	1.1710 80960 45464 16345
0.455	2.7668 09791 51767 62063	0.655	1.2441 17500 44217 10320
0.460	2.8308 08135 79022 77355	0.660	1.3242 21966 09272 13217
0.465	2.8976 80341 30803 93453	0.665	1.4122 86108 91546 86289
0.470	2.9676 00851 19895 44266	0.670	1.5093 39717 45032 78137
0.475	3.0407 57402 07324 89247	0.675	1.6165 75923 36752 73688
0.480	3.1173 52238 87305 07979	0.680	1.7353 81876 25083 27412
0.485	3.1976 03459 08084 14295	0.685	1.8673 76083 95332 79945
0.490	3.2817 46502 01613 33942	0.690	2.0144 54064 23936 11785
0.495	3.3700 35801 10110 86282	0.695	2.1788 44423 09563 08089
0.500	3.4627 46619 45506 36115	0.700	2.3631 78053 18030 01521
0.505	3.5601 77092 03883 88493	0.705	2.5705 73965 00973 09565
0.510	3.6626 50500 87897 40456	0.710	2.8047 46289 77911 85397
0.515	3.7705 17813 73417 16517	0.715	3.0701 38402 91171 97210
0.520	3.8841 60521 01533 40182	0.720	3.3720 91992 96944 66928
0.525	4.0039 93810 94402 30931	0.725	3.7170 61441 46289 59801
0.530	4.1304 70128 96286 76945	0.730	4.1128 87339 36643 00605
0.535	4.2640 83174 45098 67317	0.735	4.5691 47714 83778 14583
0.540	4.4053 72396 03473 35446	0.740	5.0976 12115 07565 15037
0.545	4.5549 28056 44360 59496	0.745	5.7128 32845 99353 60222

TABLE 2 (continued)

$q$		$q$	
0.550	4.7133 96949 21371 18651	0.750	6.4329 20560 32770 26281
0.555	4.8814 88862 91381 80462	0.755	7.2805 69678 40417 36140
0.560	5.0599 83904 35798 43329	0.760	8.2844 25342 16356 23965
0.565	5.2497 40810 95540 88729	0.765	9.4809 21549 98351 20543
0.570	5.4517 06404 51862 65657	0.770	(2) 1.0916 77562 51044 47580
0.575	5.6669 26365 22168 57785	0.775	1.2652 40625 92913 19610
0.580	5.8965 57535 84756 08563	0.780	1.4766 66519 75762 88379
0.585	6.1418 82004 06658 23969	0.785	1.7363 45867 23513 64940
0.590	6.4043 23255 57329 03586	0.790	2.0581 12017 03871 80290
0.595	6.6854 64745 11986 20930	0.795	2.4605 84021 46404 05534
0.800	(2) 2.9691 15980 84233 49162	0.900	(5) 7.7756 42033 59582 18102
0.805	3.6186 65777 36073 96899	0.905	(6) 1.8006 49752 17839 11617
0.810	4.4580 79294 33274 70763	0.910	4.5845 54975 27556 60330
0.815	5.5565 89752 24798 85188	0.915	(7) 1.3050 99381 12968 62099
0.820	7.0138 44490 94232 38569	0.920	4.2423 11062 36353 51447
0.825	8.9756 54873 32449 93411	0.925	(8) 1.6171 47804 94775 34530
0.830	(3) 1.1659 21959 77285 21165	0.930	7.4812 80658 77032 91079
0.835	1.5394 37070 41432 17384	0.935	(9) 4.3930 25319 70036 30536
0.840	2.0692 55587 49933 90635	0.940	(10) 3.4761 85841 88291 87124
0.845	2.8365 05368 37927 38223	0.945	(11) 4.0221 00106 84617 59589
0.850	3.9730 98640 63737 13114	0.950	(12) 7.6292 45417 67438 93757
0.855	5.6994 02411 43760 85755	0.955	(14) 2.7987 81240 16028 87338
0.860	8.3946 34496 25568 41446	0.960	(16) 2.5447 20788 43043 04611
0.865	(4) 1.2733 01776 91168 92024	0.965	(18) 8.4696 36897 34482 06597
0.870	1.9957 26217 95492 07005	0.970	(22) 4.9772 95738 72751 97198
0.875	3.2451 27665 78675 43951	0.975	(27) 1.0444 71050 49055 46476
0.880	5.4996 22126 39070 63228	0.980	(34) 1.3005 98028 90543 75339
0.885	9.7670 58659 86586 96614	0.985	(45) 9.0771 11377 36256 08344
0.890	(5) 1.8294 25486 47650 80208	0.990	(69) 4.8151 08834 54340 23449
0.895	3.6417 39645 78159 87095	0.995	(140) 9.3436 10573 31744 65727

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